NASA
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Goddard Space Flight Center
Contract No. NAS 5-2078

NASA TT F-8622

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ff 653 July 65

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SIMPLEST PRECISE SOLUTIONS OF THE BOLTZMANN EQUATION FOR RAREFIED GAS FLOWS [ ]

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by

A. A. Nikol'skiy (USSR) Aug. 1963 7p refs

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N66 29420 (ACCESSION NUMBER)	(THRU)
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(NASA CR OR TMX OR AD NUMBER)	12 (CATEGORY)

[redacted]

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CONSULTANTS AND DESIGNERS, INC. from Doklady Akad. Nauk SSSR
Arlington, Virginia
August 1963

(Moscow), v. 151, no. 2, 1963 p 299-301

NASA TT F-8622

DRAFT TRANSLATION ST-AGD-10039

AUG 29 1963

SIMPLEST PRECISE SOLUTIONS OF THE BOLTZMANN EQUATION FOR RAREFIED GAS FLOWS *

Doklady A. N.SSSR,
Aerodinamika,
Tom 151, No. 2, 299-301,
Izd-vo A. N.SSSR, 1963

by A. A. Nikol'skiy

Considered in this paper are the flows of a monatomic gas or of monatomic gas mixtures in an infinite three-dimensional space, for which the distribution functions of particles' natu al velocities is identical for all points. Without arguing this point, we shall estimate everywhere the time, spatial coordinates and velocities as dimensionless, relating them respectively to same time t_0, length l_0 and velocity l_0/t_0 scales for all cases. We shall designate by x, y, z, t the Descartes coordinates and the time and by u, v, w — the particle velocities; c shall be the vector with components — u, v, w. We shall understand for the distribution function

$$f_i\,(t,\ x,\ y,\ z,\ u_i,\ v\ ,\ w_i)$$

the number of particle of the i-th kind at the point x, y, z, u_i, v_i, w_i of a six-dimensional space, related to volume element $dx\,dy\,dz\,du_i dv_i dw_i$.

The Boltzmann equation has the form [1]

$$\frac{\partial f_i}{\partial t} + u_i \frac{\partial f_i}{\partial x} + v_i \frac{\partial f_i}{\partial y} + w_i \frac{\partial f_i}{\partial z} = I_i\,(t,\ r,\ c_i), \qquad (1)$$

where I_i are the integrals of collisions:

$$I_i = \sum_j \iiint [f_i\,(t,\ r,\ c_i')\, f_j\,(t,\ r,\ c_j') - f_i\,(t,\ r,\ c_i)\, f_j\,(t,\ r,\ c_j)]\, g_{ij}\, b\, db\, d\varepsilon\, dc_j. \qquad (2)$$

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* [PROSTEYSHIYE TOCHNYYE RESHENIYA URAVNENIYA BOL'TSMANA DLYA DVIZHENIYA RAZREZHENNOGO GAZA]

[redacted]

Here $\mathbf{r}$ is a vector with components x, y, z; c_i is a vector with components u_i, v_i, w_i; $g_{ij} = |c_i - c_j|$; b is a dimensionless distance of aim; ε is the corresponding angle, $dc_j = du_j dv_j dw_j$.

The vectors c_i', c_j' of particle velocities after collision are linked with c_i, c_j, determined by the character of interaction by the functional dependence:

$$c_i' = \varphi_{ij}(c_i, c_j, b, \varepsilon), \quad c_j' = \psi_{ij}(c_i, c_j, b, \varepsilon). \tag{3}$$

It is obvious that when particles are hard elastic spheres, the following similitude property takes place:

$$\varphi_{ij}(\lambda c_i, \lambda c_j, b, \varepsilon) = \lambda\varphi_{ij}(c_i, c_j, b, \varepsilon); \quad \psi_{ij}(\lambda c_i, \lambda c_j, b, \varepsilon) = \lambda\psi_{ij}(c_i, c_j, b, \varepsilon), \tag{4}$$

where λ is a positive arbitrary quantity.

It is assumed below, that for each transition to new variables in the left-hand part of equations (1) there is an identical transition in the integrals in the integrals I_i too.

THREE-DIMENSIONAL SCATTERING-GATHERING. Assume that the macroscopic velocities are distributed according to the law

$$u = \frac{x}{t}, \quad v = \frac{y}{t}, \quad w = \frac{z}{t}. \tag{5}$$

For the case of a nonrarefied gas these flows were obtained by L. I. Sedov [2]. The last expressions characterize two types of flows. Considering them at $0 < t < \infty$, we obtain the scatter, and at $-\infty < t < 0$ — the gathering. We shall require the fulfillment of the equalities

$$f_i(t, x, y, z, u_i, v_i, w_i) = f_i(\tau, 0, 0, 0, U_i, V_i, W_i),$$

where $\tau = t$, $U_i = u_i - x/t$, $V_i = v_i - x/t$, $W_i = w_i - z/t$; U_i, V_i, W_i — are the natural velocities of particles. With variables τ, U_i, V_i, W_i the equations (1) will take the form

$$\frac{\partial f_i}{\partial \tau} - \frac{1}{\tau}\left(U_i\frac{\partial f_i}{\partial U_i} + V_i\frac{\partial f_i}{\partial V_i} + W_i\frac{\partial f_i}{\partial W_i}\right) = I_i. \tag{6}$$

At $x = y = z = 0$ we have $U_i = u_i, V_i = v_i, W_i = w_i$, and thus the equations (6) are equations for the distribution functions of absolute velocities at the point $x = y = z = 0$. This is precisely the way we shall understand them below.

Let us still introduce the variables $T = \tau$, $\xi_i = \tau U_i$, $\eta_i = \tau V_i$, $\zeta = \tau W_i$. With these variables the equations (6) will take the form

$$\frac{\partial f_i}{\partial T} = I_i. \tag{7}$$

In the trivial case when the integral I_i is identically equal to zero, $\partial f_i/\partial T = 0$, and we have

$$f_i = F_i(\xi_i, \eta_i, \zeta_i) = F_i(tU_i, tV_i, tW_i), \tag{8}$$

where F_i are arbitrary functions of arguments.

The total number of particles of i-th kind in the volume t^3 is

$$|t^3| \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_i(tU_i, tV_i, tW_i)\, dU_i dV_i dW_i = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_i(a, b, c)\, da\, db\, dc. \tag{9}$$

It is invariable in time, as must be. The kinetic self-energy of particles of i-th kind in the same volume is equal to

$$|t^3| \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_i(tU_i, tV_i, tW_i) \frac{U_i^2 + V_i^2 + W_i^2}{2} dU_i dV_i dW_i =$$

$$= \frac{1}{t^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_i(a, b, c) \frac{a^2 + b^2 + c^2}{2} da\, db\, dc. \tag{10}$$

It varies inversely-proportionally to t^2. The integrals I_i are identically equal to zero in trivial case of absence of interaction in the case when the distribution of particle velocities is at each moment of time Maxwellian. Bearing in mind the last case, we shall seek the solution of the system of equations (7) in the form

$$f_i = \alpha_i \exp[k^2 m_i (\xi_i^2 + \eta_i^2 + \zeta_i^2)] = \alpha_i \exp[-k^2 m_i t^2 (U_i^2 + V_i^2 + W_i^2)], \tag{11}$$

where k, α_i are constants, m_i are the dimensionless masses of particles of the i-th kind. These expressions for distribution functions satisfy the conditions (8) and thus transform into zero the left-hand part of the equation (6), as previously. However, they also transform into zero the right-hand part of (6), for they provide at each fixed moment of time the Maxwellian velocity distributions, for which the collision integrals are zero for any potential law of interaction [1]. Bearing in mind the uniqueness of Boltzmann equations' solution in time when the distribution functions are given at a certain moment of time, we obtain :

THEOREM 1. — If at a three-dimensional scattering or gathering, marked by the equality (5), of a mixture of ideal monatomic gases the distribution of natural velocities is Maxwellian, it will also be Maxwellian at any subsequent moment of time. If at $t = t_1$

$$f_i = \alpha_i \exp\left[-k^2 m_i (U_i^2 + V_i^2 + W_i^2\right] t_i^3,$$

the equalities (11) will be valid at $t > t_1$. At the same time, and correspondingly to equalities (9) and (10), the number of particles of each kind in the volume $|t|^3$ is constant, and the total kinetic energy of all particles in this volume being proportional to the temperature, will vary inversely-proportionally to the quantity t^2.

Let us examine now a model, when particles are hard elastic spheres. We shall consider first an arbitrary uniform state of gas, rather than the flow "scattering-gathering", i.e. when the distribution functions $f_i - f_i^0$ in equations (1) do not depend on $\underline{r}$ but on $\underline{t}$:

$$\frac{\partial f_i^0(t, c_i)}{\partial t} = \sum_j \iiint \left[f_i^0(t, c_i') f_j^0(t, c_j') - f_i^0(t, c_i) f_j(t, c_j)\right] g_{ij} b\, db\, d\varepsilon\, dc_j = I_i^0(t, c_i). \tag{12}$$

We shall attempt to compare each such state with a certain flow "scattering-gathering". We shall seek the solution of equations (7) in the form:

$$f_i = f_i^0\left[\chi(T), A_i\right], \quad A_i = |T| C_i.$$

where C_i is the vector with components U_i, V_i, W_i, the function $\chi(T)$ is subject to definition. Taking advantage of similitude properties of (3) and (4) in the transformations of the right-hand part of (7), we shall obtain in place of (7) the equations:

$$\frac{d\chi(T)}{dT}\frac{\partial f_i^0(\chi, A_i)}{\partial\chi} = \frac{1}{T^4} I_i^0(\chi, A_i).$$

The equalities (12) show, that in order to solve the problem set up, we must postulate

$$f_i = f_i(t, C_i) = f_i^0\left[\left(\frac{1}{3}\frac{t}{|t|} - \frac{1}{3}\frac{1}{t^3}\right),\ |t|C_i\right],$$

In this way, to each solution $f_i = f_i^0(t, c_i)$ of the Boltzmann equations for a uniform state of elastic sphere models would correspond the solution $f_i = f_i^0\left[\left(\beta - \frac{1}{3}\frac{1}{t^3}\right), |t|\,C_i\right]$ of Boltzmann equations for the "scattering-gathering" flows. The solution for the "scattering-gathering" case, for which at $|t| = 1$ (general case because of time to scale arbitrariness $f_i = \Phi_i(C_i)$, is obtained in the form

$$\frac{d\chi(T)}{dT} = \frac{1}{T^4},\quad \chi(T) = \beta - \frac{1}{3}\frac{1}{T^3},\quad \beta = \text{const.}$$

where $f_i^0(t, c_i)$ is the solution for a uniform state with initial conditions $f_i^0(0, c_i) = \Phi_i(c_i)$. In case of scatter $t > 0$ and $t \to \infty$, we have $f_i(t, C_i = f_i^0(1/3, tC_i)$, i.e. a distribution obtained in infinite time corresponding to that already obtained for a uniform state at $t = \frac{1}{3}$.

For gathering flows $t < 0$, at $t \to -0$ $f_i(t, C_i) = f_i^0(\infty, |t|\,C_i)$. Therefore, bearing in mind the theorem 1, valid for hard elastic sphere models, and the well-known property of appraoching the Maxwellian distribution at $t \to \infty$ and a uniform state [1], we obtain:

<u>T H E O R E M 2.</u> – <u>For elastic hard sphere models in case of gathering flows, the distribution of natural velocities will either be Maxwellian all the time, or will not, generally speaking, pass into Maxwellian distribution even at $t \to \infty$; in case of gathering flows, the distribution tends to become Maxwellian at $t \to 0$.</u>

**** THE END ****

R E F E R E N C E S

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Institute of Mechanics
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USSR Academy of Sciences.

Received on
October 15, 1962

Translated by ANDRE L. BRICHANT
under Contract No. NAS-5-2078
August 28, 1963

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